Factoring Strategies: Beginning Algebra

FACTOR GUIDE

[5 Steps]

*Always begin* with these two steps:

1. Arranging the expression to be factored in descending or ascending order. **It is easier if the leading coefficient is positive**
2. Factoring out the greatest common monomial from each term (if possible)

3. *Then proceed* according to the number of terms

TWO TERMS (BINOMIAL)

3A1. Is there a difference of squares? \((a^2 - b^2)\)
   • If so, the factors are \((a + b)(a - b)\)

3A2. Is there a sum of cubes? \((a^3 + b^3)\)
   • If so, the factors are \((a + b)(a^2 - ab + b^2)\)

3A3. Is there a difference of cubes? \((a^3 - b^3)\)
   • If so, the factors are \((a - b)(a^2 + ab + b^2)\)

THREE TERMS (TRINOMIAL)

3B. Is it a perfect square trinomial? \((a^2 + 2ab + b^2)\)
   • Factor: \((a + b)^2\)

3C. Does the first variable term have a coefficient of 1?
   • If so, write \((\_)(\_)\) and list the first term of each factor.
   • Then look for two numbers that *multiply to give the product of the last term but that add to give the coefficient of the middle term* and complete your factors.

3D. If none of the above apply, follow the steps below: (see the end of this topic for process)
   • Set up a factor table.
   • Write the first and last terms in one diagonal.
   • To the right of the table write a large \(X\)
   • Write the product of the diagonal terms in the top area.
   • Write the middle term of the trinomial in the bottom area.
   • **Find two terms whose product is the top term and whose sum is the bottom term.**
   • Enter these two terms in the other diagonal.
   • Factor the rows and columns.
   • Write all factors in your answer.
***Note: There is a way to use the quadratic formula instead of going through the steps in 3D. See end of this topic for details!

FOUR TERMS

3E. Set up a factor table.
   • Write the first and last terms in one diagonal
   • Write the two middle terms in the other diagonal.
   • ***Check to be sure the cross products are equal. If they are not, you can’t factor!
   • Factor the rows and columns and write the factors in correct form.

4. Continue until each individual factor is prime.

5. Check your results by multiplying.

Factoring with the Quadratic Formula

If a trinomial is in the quadratic form \( ax^2 + bx + c = 0 \) you can use \( a, b, \) and \( c \) and the quadratic formula to get your factors.
   • Begin by using \( a, b, \) and \( c \) to solve each of the following equations, leaving answers in improper fraction form (no decimals).

\[
\begin{align*}
    x &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\
\end{align*}
\]

NOTE: If is not a perfect square, then the trinomial is not factorable.

• Then set each equation equal to 0 to get the actual factors.

   If \( x = \frac{3}{5} \) then \( 5x = 3 \) and \( 5x - 3 = 0 \)

   If \( x = \frac{-2}{3} \) then \( 3x = -2 \) and \( 3x + 2 = 0 \)

So the factors are \((5x - 3)\) and \((3x + 2)\)

An EXAMPLE OF THE TABLE METHOD described on the front side of this handout.

Factor \( 3x^2 + 4x - 4 \)

1. Start with a 2×2 table.

2. Write the term of highest degree in the upper left corner and the lowest in the lower right.

3. Find the product of these two terms: \( (3x^2)(-4) = -12x^2 \)

4. Now, find two factors of \(-12x^2\) whose sum is the middle term \(4x\).

<table>
<thead>
<tr>
<th>Factors</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-x)(12x))</td>
<td>(-x + 12x = 11x)</td>
</tr>
<tr>
<td>((-2x)(6x))</td>
<td>(-2x + 6x = 4x)</td>
</tr>
</tbody>
</table>
5. Write these two terms in the other diagonal.

6. Factor the Greatest Common Factor from each row and column. ***Double check signs by multiplication!

<table>
<thead>
<tr>
<th>3x^2</th>
<th>-2x</th>
</tr>
</thead>
<tbody>
<tr>
<td>6x</td>
<td>-4</td>
</tr>
</tbody>
</table>

7. Finally, write the two factors as a product: 

\[3x^2 + 4x - 4 = (3x - 2)(x + 2)\]