Dimensional Analysis and Conversions

Dimensional analysis is a method by which you can change a value in one type of unit to another. You may not realize it, but you do this all of the time without thinking about it. You might figure out that a twelve ounce steak actually weighs three-fourths of a pound, or someone who is five feet tall is also 60 inches tall. You relate these different units by using **<u>Conversion Factors</u>**. A conversion factor is simply a fraction that relates two different units. Some examples are shown below.

$$\frac{12\,in}{ft} \qquad \frac{16\,oz}{lb}$$

Your objectives contain a list of the conversions which you are expected to know for the class. In particular, it is very important to know those which relate the metric and English systems of measurement. You will need at least one each for length, mass, and volume calculations. These conversations are

$$\frac{2.20\,lb}{kg} \qquad \frac{454\,g}{lb} \qquad \frac{2.54\,cm}{in} \qquad \frac{1.06\,qt}{L} \qquad \frac{946\,mL}{qt}$$

Like every conversion factor, each of these is equal to one, since the numerators and denominators are equal. To convert on unit into another, you simply multiply the value by a conversion factor which contains both the unit you are starting with and the unit you want to get to. Since conversion factors are equal to one, you don't change the value of your original measurement, only the units. For example, let's change my weight of 161 lb into kg. We will need a conversation factor that relates these two units, which is 2.20 lb = 1 kg. Thus,

$$161 \text{ lb} \times \frac{1\,kg}{2.20\,lb} = 73.2 \text{ kg}$$

The units of lb in the numerator of my weight and the denominator of the conversion factor cancel, leaving the desired unit of kg. Suppose that you wanted to determine how many dollars there would be in 3475 nickels. We would then use the conversion of $1 = 20 nickels.

$$3475 \text{ nickels} \times \frac{\$1}{20\,nickel} = \$173.75$$

Again the units of nickels cancel, leaving the desired unit of dollars. One question you might be asking is how do you know which unit to put on the top of the conversion factor? One can just as easily write 20 nickels over $1 or 2.20 lb over 1 kg. The answer is quite simple. Whatever unit is in the initial measurement must go in the denominator of the conversion factor. In this way the units will cancel. If you had inadvertently inverted the conversion and placed the units of nickels in the numerator, the units would not have cancelled and the units in your answer would have been nickels squared over dollars, which would make no sense.

It is often the case (always on quizzes and exams) that a problem will require more than one conversion. For example, suppose we wanted to determine how many pints are equal to 13.5 L. We have not learned a conversion that would relate these two units, so we must use two conversion factors to connect pints and liters.

$$13.5 \text{ L} \times \frac{1.06 qt}{1 L} = 14.31 \text{ qt}$$

$$14.31 \text{ qt} \times \frac{2 pt}{1 qt} = \underline{\mathbf{28.6\ pt}}$$

The only difference between this problem and the first two is that we must do two calculations instead of one. These multi-step problems can be simplified by linking the steps together rather than doing two separate calculations, as shown below. In this case the liters still cancel as do the quarts, leaving the desired unit of pints.

$$13.5 \text{ L} \times \frac{1.06 qt}{1 L} \times \frac{2 pt}{1 qt} = \underline{\mathbf{28.6\ pt}}$$

In some cases it may take several conversions to complete a problem. In this case it is useful to write down a **Unit Plan**, which shows the units which will be used to connect the initial and final values in the problem. For example, suppose we wish to determine the distance across Nebraska, 465 miles, in millimeters. In this case will need to connect miles and millimeters. Since miles is an English unit and mm is metric, the conversion of 2.54 cm = 1 in will be needed. However, it cannot be used until miles is first changed into inches. Once this is done, inches can changed to cm, and using the metric conversations you learned, cm will be converted to m and then to mm.

mi — ft — in — cm — m — mm

Unit Plan

Following the unit plan, it is easy to see which unit to use next. Always place the unit you are coming from in the denominator of the next conversion and the unit you are going to on the top. The problem will be set up as follows.

$$465 \text{ mi} \times \frac{5280\,ft}{mi} \times \frac{12\,in}{ft} \times \frac{2.54\,cm}{in} \times \frac{1\,m}{100\,cm} \times \frac{1000\,mm}{m} = \underline{\mathbf{7.48 \times 10^{8}\ mm}}$$

As the old saying goes, "a journey of a thousand miles begins with a single step", so it is with conversation problems. Even a problem such as the previous one is simply a series of simple one-step conversation problems linked together, so don't let it overwhelm you.

Density

As you have already learned, density is simply the mass of an object divided by its volume. Thus density can be used to relate the mass of something with its volume or vice-versa. While this can be done using algebra, that approach often sends students into a state of abject terror. So instead, we will treat density as simply another conversion. For example, suppose we have an iron nail that weighs 5.83 g and we want to know the volume of the nail. Given that the density of iron is 7.87 g/mL, we have

$$5.83 \text{ g} \times \frac{1\,mL}{7.87\,g} = \underline{\mathbf{0.741\ mL}}$$

As always, the unit you are going to is in the numerator and unit you started with is in the denominator. Density can also be easily used in multi-step problems. For example, how many pounds would 1.25 gallon of mercury metal weigh, given its density of 13.6 g/ml? Before we can use the density, the gallons must be converted to mL. Then mL can be converted into g and on to lb. Thus our unit plan would be

ga — qt — mL — g — lb

$$1.25 \text{ ga} \times \frac{4\,qt}{ga} \times \frac{946\,mL}{qt} \times \frac{13.6\,g}{mL} \times \frac{1\,lb}{454\,g} = \underline{\mathbf{142\ lb}}$$

Using these examples, you should now be able to work on the problems in the first two worksheets.