MATH 03 <u>Study Questions for The Final Exam</u>

Chapter 12.

1. Which of the following statements is true about the vectors

   $\mathbf{u} = -\frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{1}{4}\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} - \frac{4}{3}\mathbf{j} + \mathbf{k}$?

   a. **u** and **v** are parallel. b. **u** is a unit vector of **v**.

   c. The angle between **u** and **v** is $\frac{\pi}{4}$ d. **u** and **v** are orthogonal.

2. Which of the following does **<u>NOT</u>** imply that two nonzero vectors are parallel?

   a. $\vec{v} \times \vec{w} = \vec{0}$ b. $\vec{v} = -2\vec{w}$ c. $proj_{\vec{w}}\vec{v} = \vec{v}$ d. $\vec{v} \cdot (\vec{v} \times \vec{w}) = 0$

3. Let $\vec{v} \cdot (\vec{u} \times \vec{w}) = 3$. Find $\vec{w} \cdot (\vec{u} \times \vec{v})$

   a. 3 b. 0 c. $\sqrt{3}$ d. -3

4. Which of the following does **<u>NOT</u>** imply that the volume of the parallelepiped having $\vec{u}, \vec{v}, \vec{w}$ as adjacent sides is zero?

   a. $\vec{u}, \vec{v}, \vec{w}$ all lie in the same plane

   b. $\vec{u} \cdot \vec{w} = 0$ and $\vec{u} \cdot \vec{v} = 0$

   c. $\vec{u} = \vec{v} + \vec{w}$

   d. $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$

5. Which of the quadric standard equations can be solved for z ?

   a. Paraboloids b. Hyperboloids c. Ellipsoid d. Cone

6. Write an equation of the plane that contains the line given by $x = t, y = 1 + 3t, z = -1 + 2t$ and is perpendicular to the line $x = 1 - 17t, y = -5 + t, z = 3 + 7t$.

   a. 17x - y - 7z + 1 = 0

   b. x + 3y + 2z + 8 = 0

   c. 17x - y - 7z - 6 = 0

   d. x + 3y + 2z - 1 = 0

7. Change the spherical $(6, -\frac{\pi}{6}, \frac{\pi}{3})$ coordinates to cylindrical.

   a. $(\frac{3}{2}, -\frac{3\sqrt{3}}{2}, 3)$ b. $(-\frac{3\sqrt{3}}{2}, \frac{3}{2}, 3)$

   c. $(3\sqrt{3}, 3, \frac{\pi}{3})$ d. $(3\sqrt{3}, -\frac{\pi}{6}, 3)$

8. Find an equation in cylindrical coordinates for the surface given by the rectangular equation $x^2 - y^2 = z$.

   a. $r = \cot\varphi \csc\varphi$ b. $r^2 = z \sec 2\theta$

   c. $r = \cot\varphi \csc\varphi \sec 2\theta$ d. $r^2 = z$

Chapter 13.

9. Find a set of parametric equations for the line tangent to the helix $\mathbf{r}(\mathrm{t}) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + t\,\mathbf{k}$ at the point corresponding to $t = \pi/6$.

   a. $x = \sqrt{3} + s, \quad y = 1 - \sqrt{3}\,s, \quad z = \dfrac{\pi}{6} + s$

   b. $x = -s, \quad y = \sqrt{3}\,s, \quad z = s$

   c. $x = \sqrt{3} - s, \quad y = 1 + \sqrt{3}\,s, \quad z = \dfrac{\pi}{6} + s$

   d. $x = -1 + \sqrt{3}\,s, \quad y = \sqrt{3} + s, \quad z = 1 + \dfrac{\pi}{6}s$

10. Which of the following implies that $\vec{r}(t)$ is arc length parametrization ?

    a. $\int \|\vec{r}\,'(t)\|\,dt = 1$ b. $\|\vec{r}\,'(t)\| = 1$ for all t

    c. $\int_0^t \|\vec{r}\,'(u)\|\,du = 1$ d. $\|\vec{r}(t)\| = 1$ for all t

11. Find the arc length function for $\mathbf{r}(\mathrm{t}) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + t\,\mathbf{k}$.

    a. $s = \sqrt{6}\,t$ b. $s = t$ c. $s = 3t$ d. $s = \sqrt{10}\,t$

12. Find the principal unit normal vector for the curve $\mathbf{r}(\mathrm{t}) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + t\,\mathbf{k}$.

    a. $\sin t\,\mathbf{i} + \cos t\,\mathbf{j}$ b. $-\sin t\,\mathbf{i} + \cos t\,\mathbf{j}$

    c. $-\sin t\,\mathbf{i} - \cos t\,\mathbf{j}$ d. $\sin t\,\mathbf{i} - \cos t\,\mathbf{j}$

13. Calculate the curvature of $\boldsymbol{r}(t) = \dfrac{1}{3}t^3\,\mathbf{i} + \dfrac{\sqrt{2}}{2}t^2\,\mathbf{j} + t\,\mathbf{k}$ at the point where $t = 2$.

    a. $\dfrac{2\sqrt{13}}{125}$ b. 4/5 c. $\dfrac{\sqrt{2}}{25}$ d. $\dfrac{\sqrt{2}}{(t^2+1)^2}$

14. An object moves according to the position function $\mathbf{r}(\mathrm{t}) = -2\cos 3t\,\mathbf{i} + \sin 3t\,\mathbf{j}$.
    Find the maximum speed of the object.

    a. 6 b. 12 c. 3 d. 0

15. Which of the following does **<u>NOT</u>** imply that $a_T(t) = 0$?

    a. $\vec{v}(t)$ and $\vec{a}(t)$ are orthogonal

    b. $\vec{v}(t) = \vec{0}$

    c. $\vec{a}(t) = \vec{0}$

    d. $\|\vec{v}\,'(t)\|' = 0$

16. An object moves with constant speed of $\sqrt{14}$ along the circle of radius 7 centered at the origin. Find the vector normal component of the acceleration when the object is at the highest point.

    a. $\sqrt{14}\,(\mathbf{i} + \mathbf{j})$ b. -7**j** c. -2**j** d. 2**i**

17. Find the limit: $\lim_{(x,y)\to(2,-1)}\left(\frac{x^2-4y^2}{x+2y}\right)$

a. 0 b. 4
c. 5 d. -2

18. Under what conditions $f_{xy}(x_0,y_0) = f_{yx}(x_0,y_0)$ holds?

a. If $f(x,y)$ is differentiable at (x_0,y_0)
b. Always
c. Both second partials exist near (x_0,y_0) and are continuous
d. Both first partials exist near (x_0,y_0) and are continuous

19. Find parametric equations for the normal line to the surface $z = 3yx^2 - y^2$ at the point (-2,1,11).

a. $x = -2 - t,\ y = 1 + 12t,\ z = 11 - t$ b. $x = -2 - 12t,\ y = 1 + 10t,\ z = 11 - t$
c. $x = -2 + t,\ y = t,\ z = t$ d. $x = -2 + 12t,\ y = 1 + 10t,\ z = 11 - 2t$

20. Find the points on the surface $x^2 + y^2 + 2x - 4y + z^2 + 1 = 0$ where the tangent plane is horizontal.

a. (1, -2, 2), (1, -2, -2) b. (-1, 2, -2)
c. (-1, 2, 2), (-1, 2, -2) d. (-1, 2, 4), (-1, 2, -4)

21. Find the directional derivative of $f(x, y) = x^2y$ at the point (1, -3) in the direction of $-2\mathbf{i} + \mathbf{j}$.

a. 13 b. $-11/\sqrt{5}$ c. $13/\sqrt{5}$ d. -11

22. Which of the following is **NOT** a true statement about $\nabla f(x_0,y_0)$?

a. If $f_x(x_0,y_0) = f_y(x_0,y_0) = 0$, then $\nabla f(x_0,y_0) = \vec{0}$
b. If $\nabla f(x_0,y_0) = \vec{0}$, then any directional derivative is 0 at (x_0,y_0)
c. If $D_{\vec{v}}\ f(x_0,y_0) = D_{\vec{w}}\ f(x_0,y_0) = 0$ in two nonparallel directions $\vec{v}$ and $\vec{w}$, then $\nabla f(x_0,y_0) = \vec{0}$
d. If $\max_{\vec{v}} D_{\vec{v}}\ f(x_0,y_0) = 1$, then $\|\nabla f(x_0,y_0)\| = 1$
e. $\nabla f(x_0,y_0)$ is parallel to the level curve of $f(x,y)$ through (x_0,y_0)

23. Which of the following implies that a continuous on the region D function $f(x,y)$ attains an absolute maximum on D ?

a. $D = \{x^2 + y^2 < 2\}$ b. $D = \{x + y > 2\}$
c. $D = \{xy \geq 0\}$ d. $D = \{0 \leq x, y \leq 5\}$

24. Determine relative extrema and saddle points of $z = x + y - 1/xy$

a. Minimum: (1, 1, 1) b. Maximum: (1, 1, 1)
c. Minimum: (-1, -1, -3) d. Maximum: (-1, -1, -3)

Chapter 15.

25. Evaluate the integral $\iint_R \frac{x}{\sqrt{1+y^2}}\,dA$ where R is the region in the first quadrant bounded by the graphs of $y = x^2$, y = 4 and x = 0.

a. 4($\sqrt{17}$ - 1) b. 68 $\sqrt{17}$ c. 1/2($\sqrt{17}$ - 1) d. $\sqrt{17}$ -1

26. $\int_0^2 \int_1^{e^y} f(x,y)\,dx\,dy$ is the same as

a. $\int_1^{e^y} \int_0^2 f(x,y)\,dx\,dy$ b. $\int_1^{e^y} \int_0^2 f(x,y)\,dy\,dx$

c. $\int_1^{e^2} \int_{\ln x}^2 f(x,y)\,dy\,dx$ d. $\int_e^{e^2} \int_{\ln x}^{\ln 2} f(x,y)\,dy\,dx$

27. Find the area enclosed by the graph of $r = 1 - \cos\theta$.

a. $\frac{3}{2}\pi$ b. $\frac{5}{4}\pi$ c. 2π d. $\frac{3}{4}\pi$

28. Find the surface area of the portion of the surface $z = 16 - x^2 - y^2$ above the plane $z = 7$.

a. $\frac{\pi}{6}(37^{3/2} - 1)$ b. $\frac{\pi}{12}(37^{3/2} - 1)$ c. $\frac{\pi}{6}37^{3/2}$ d. $\frac{\pi}{6}\sqrt{37}$

29. Use a triple integral to find the volume of the solid in the first octant bounded by the coordinate planes, the plane $2x + y = 2$ and the paraboloid $z = 4 - x^2 - y^2$.

a. 17/3 b. 19/6 c. 19/3 d. 17/6

30. Find the limits of integration for calculating the volume of the solid Q enclosed by the graphs of $x = y^2$, z = 0 and $x + z = 1$.

a. $\int_0^{y^2} \int_{-\sqrt{x}}^{\sqrt{x}} \int_0^{1-x} dz\,dy\,dx$ b. $\int_0^1 \int_0^{\sqrt{x}} \int_0^{1-x} dz\,dy\,dx$ c. $\int_0^1 \int_0^1 \int_0^1 dz\,dy\,dx$ d. $\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \int_0^{1-x} dz\,dy\,dx$

31. Find $\iint_R e^{xy}\,dA$ where R is enclosed by $y = x,\ y = \frac{x}{2},\ y = \frac{1}{x},\ y = \frac{2}{x}$

a. $\frac{1}{2}(e^2 - e)\ln 2$ b. $\frac{1}{2}e^2 \ln 2$ c. $e^2 \ln 2$ d. $\ln 2$

32. What change of variables is likely to simplify $\iint_R (x^2 - y^2)e^{x+y}\,dA$?

a. $T(u,v) = (u^2 - v^2, u+v)$ b. $T(x,y) = (x^2 - y^2, x+y)$

c. $T(u,v) = (u+v, uv)$ d. $T(u,v) = ((u+v)/2, (v-u)/2)$

Chapter 16.

33. Find $\int_C (x+2y)ds$, C is counterclockwise upper half of the unit circle.
a. -2 b. 4 c. 0 d. -4

34. Find $\int_C ydx + xdy + z^2dz$ for the curve $\mathbf{r}(t) = t\mathbf{i} + \cos t\,\mathbf{j} + \sin t\,\mathbf{k},\quad 0 \le t \le 2\pi.$
a. $1 - \pi$ b. 2π c. $\pi - 1$ d. -2π

35. $\int_{(1,1)}^{(-1,0)} (y^2 - 3x^2)dx + (2xy + 2)dy =$ a. -1 b. -2 c. 0 d. 1

36. Let f and g be continuous and have continuous partial derivatives on some open, simply connected region D. Which of the following implies that $\mathbf{F} = f\mathbf{i} + g\mathbf{j}$ is conservative in D?

a. $\oint_C \mathbf{F}\cdot d\mathbf{r} \neq 0$ for every piecewise smooth closed curve C in D

b. $\int_C \mathbf{F}\cdot d\mathbf{r}$ is independent of path for every closed piecewise smooth curve C in D

c. $\mathbf{F} = \operatorname{div} \varphi$ for some differentiable in D function $\varphi(x,y)$ d. $f_y = g_x$ at each point in D

37. C is a path from (0, 0) to (1, 0) to (1, 1) to (0, 1) and back to (0, 0). Then, $\oint_C 5ydx - 8xdy =$

a. $\int_0^1\int_0^1 (-8-5)\,dx\,dy$ b. $\int_0^1\int_0^1 (5-(-8)\,dx\,dy$ c. $\int_0^1\int_0^1 (-8x-5y)\,dx\,dy$ d. $\int_0^1\int_0^1 -40xy\,dx\,dy$

38. R is the part of the unit circle in Quadrant III and C is the boundary of R oriented counterclockwise. Which of the following does **NOT** represent the area of R?

a. $\iint_R dA$ b. $\oint_C x\,dy$ c. $-\oint_C y\,dx$ d. $\frac{1}{2}\oint_C x\,dy + y\,dx$ e. $\int_{3\pi/2}^{\pi}\int_0^1 r\,dr\,d\theta$

39. Flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through $z = \sqrt{1 - x^2 - y^2}$ is a. $\pi/2$ b. π c. $3\pi/2$ d. 2π

40. Let G be a solid with surface σ, $\mathbf{F}$ continuous with continuous partial derivatives.
Which of the following does **NOT** imply that the outward flux of $\mathbf{F}$ through σ is 0?
a. div $\mathbf{F} = \mathbf{0}$ at every point in G b. $\mathbf{F}$ is tangent to σ at every point
c. σ is given by $H(x,y,z) = 0$ and $\mathbf{F}\cdot\nabla H = 0$ at every point d. $\mathbf{F}$ points outwards at every point on σ

41. Find the outward flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through σ: $1 = x^2 + y^2,\ 0 \le z \le 1.$
a. 1 b. 0 c. 3π d. 6π

42. Let σ be the surface of the solid G and $\mathbf{F} = 2x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$. Then, $\oiint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS =$

a. 0 b. $\iiint_G (x+y+z)dS.$ c. 3Volume(G) d. Area of σ

43. Let σ be a smooth oriented surface bounded by a simple, closed, smooth curve C, $\mathbf{F}$ continuous with continuous partials. Which of the following does **NOT** imply that the work done by $\mathbf{F}$ is 0?
a. curl $\mathbf{F} = \vec{\mathbf{0}}$ at every point on σ b. $\mathbf{F}$ is normal to C at every point
c. div $\mathbf{F} = \mathbf{0}$ at every point on σ d. $\mathbf{F} = \nabla\varphi$ for some differentiable function $\varphi(x,y,z)$
e. Circulation density in any direction is zero on σ

44. C is the triangle from (0, 0, 0) to (2, 0, 0) to (0, 2, 1) to (0, 0, 0), which lies in the plane $z = y/2$.
Using Stokes's Theorem $\oint_C (-3y^2\mathbf{i} + 4z\mathbf{j} + 6x\mathbf{k})\cdot d\mathbf{r} =$ a. 2 b. 2/3 c. 14 d. 0

MATH 03 Answers to Study Questions for The Final Exam

1. a
2. d
3. d
4. b
5. a
6. c
7. d
8. b
9. c
10. b
11. d
12. c
13. c
14. a
15. d
16. c
17. e
18. c
19. b
20. c
21. c
22. e
23. d
24. d
25. c
26. c
27. a
28. a
29. b
30. d
31. a
32. d
33. b
34. b
35. a
36. d
37. a
38. d
39. d
40. d
41. c
42. a
43. c
44. c