

**Chapter 15 Concept Check**

Find Answers in the book, pay special attention to

**Theorems 15:** 2.2, 3.3, 5.2, 8.2

1. For a simple polar region  $R$ ,  $\int_R f(r, \theta) dA$  equals

a.  $\int_{\lambda}^{\beta} \int_{r_1(\theta)}^{r_2(\theta)} f(r, \theta) dr d\theta$       b.  $\int_{\lambda}^{\beta} \int_{r_1(\theta)}^{r_2(\theta)} r f(r, \theta) d\theta dr$

c.  $\int_{\lambda}^{\beta} \int_{r_1(\theta)}^{r_2(\theta)} r f(r, \theta) dr d\theta$       d.  $\int_{r_1(\theta)}^{r_2(\theta)} \int_{\lambda}^{\beta} r f(r, \theta) dr d\theta$

2.  $\int_0^2 \int_1^{e^y} f(x, y) dx dy$  is the same as

a.  $\int_1^{e^2} \int_0^2 f(x, y) dx dy$       b.  $\int_1^{e^2} \int_0^2 f(x, y) dy dx$

c.  $\int_1^{e^2} \int_{\ln x}^2 f(x, y) dy dx$       d.  $\int_e^{e^2} \int_{\ln x}^{\ln 2} f(x, y) dy dx$

Answer c.

3.  $\int_0^9 \int_0^{3-\sqrt{x}} \int_0^z f(x, y, z) dy dz dx$  is the same as

a.  $\int_0^9 \int_0^{3-\sqrt{x}} \int_y^{3-\sqrt{x}} f(x, y, z) dz dy dx$       b.  $\int_0^9 \int_0^{3-\sqrt{x}} \int_z^{3-\sqrt{x}} f(x, y, z) dz dy dx$

c.  $\int_0^9 \int_z^{3-\sqrt{x}} \int_y^{3-\sqrt{x}} f(x, y, z) dz dy dx$       d.  $\int_0^9 \int_0^{3-\sqrt{x}} \int_0^{3-\sqrt{x}} f(x, y, z) dz dy dx$

Answer a.

4. Surface area of a parametric surface  $\sigma$  over  $u, v$  region  $R$  equals

a.  $\int_R \|\vec{r}_u \times \vec{r}_v\| dA$       b.  $\int_R \|\vec{r}_{uv} \times \vec{r}_{vu}\| dA$

c.  $\int_R \vec{r}_u \times \vec{r}_v dA$       d.  $\int_R \|\vec{r}_u \cdot \vec{r}_v\| dA$

5. What transformation will you make to simplify the integration over the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ?

a.  $T(u, v) = (au, bv)$       b.  $T(x, y) = (ax, by)$

c.  $T(u, v) = (auv, bv)$       d.  $T(x, y) = \left(\frac{x+y}{a}, \frac{x-y}{b}\right)$

MATH 03 Chapter 15 Practice Test Please, *show all work and circle final answers.*

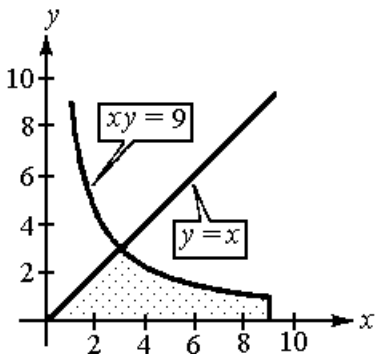
Please, *do NOT change*  $e$ ,  $\sqrt{2}$ ,  $\ln 3$  and so on, to decimals.

Please, use *exact values* for basic angles, for example, use  $\frac{\sqrt{3}}{2}$  for  $\sin \frac{\pi}{3}$ .

1. Evaluate the integral  $\int_0^{\sqrt{\pi}} \int_{\pi/6}^{y^2} 2y \cos x \, dx \, dy$ .

2. Evaluate the integral  $\int_0^1 \int_y^1 \sin x^2 \, dx \, dy$ . *Hint: Reverse the order of  $\int$ .*

3. Write  $dydx$  integrals to calculate the area of the region shown.



4. Evaluate the integral  $\iint_R (x^2 + 4y) \, dA$  where  $R$  is the region bounded by the graphs of  $y = 2x$  and  $y = x^2$ .

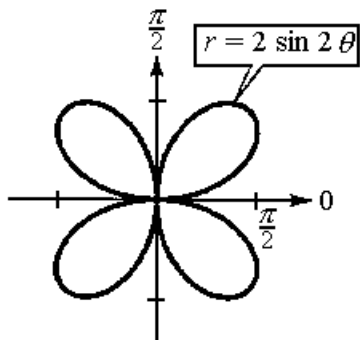
5. Let  $R$  be the region bounded by the graphs of  $y = x$  and  $y = 3x - x^2$ . Use an iterated integral to calculate the area of the region  $R$ .

6. Use a double integral to find the volume of the solid in the first octant bounded by the plane  $y = x$ , cylinder  $z = 4 - x^2$  and  $yz$  plane.

7. Use a double integral to calculate the volume of the solid under the surface  $z(x, y) = x^2 y^2$  and above the closed region bounded by the lines  $y = 1$ ,  $y = 2$ ,  $x = 0$ , and  $y = x$ .

8. Evaluate  $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy \, dx$  by changing to polar coordinates.

9. Use polar coordinates to calculate the area of the region shown.



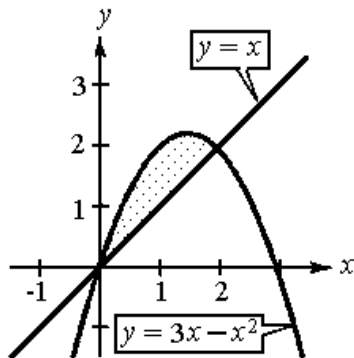
10. Use polar coordinates to find the volume of the solid under the plane  $z = 4x$  and above the circle  $x^2 + y^2 = 16$  in  $xy$  plane.
11. Find the surface area for that portion of the surface  $z = xy$  that is inside the cylinder  $x^2 + y^2 = 1$ .
12. Find the surface area of the paraboloid  $z = x^2 + y^2$  below the plane  $z = 4$ .
13. Use a triple integral to find the volume of the solid in the first octant bounded by the graphs  $z = 1 - y^2$ ,  $y = 2x$  and  $x = 3$ .
14. Find the Jacobian of the transformation  $T(u, v) = \left(\frac{u}{v}, v\right)$ .
15. Region  $R$  is bounded by  $x + y = 1$ ,  $x + y = 2$ ,  $2x - 3y = 2$  and  $2x - 3y = 5$ . Use a change of variables to find  $\iint_R (5x + 5y) dA$ .
16. Use a change of variables to find the area of  $\frac{x^2}{16} + \frac{y^2}{25} \leq 1$  in quadrant I.

Math 03 Chapter 15 Practice Test Answers.

1.  $(4 - \pi) / 2$                       2.  $(1 - \cos 1) / 2$

3.  $\int_0^3 \int_0^x dy dx + \int_3^9 \int_0^{9/x} dy dx$                       4.  $152 / 15$

5. Area =  $4 / 3$



6. 4

7.  $7 / 2$

8.  $\frac{\pi}{2}(e - 1)$

9.  $2\pi$

10.  $512 / 3$

11.  $\frac{2\pi}{3}(2\sqrt{2} - 1)$

12.  $\frac{\pi}{6}(17^{3/2} - 1)$

13.  $15 / 8$

14.  $1 / v$

15.  $9 / 2$

16.  $5\pi$