Chapter 14 Concept Check

Find Answers in the book, pay special attention to

Theorems 14: 2.2, 3.2, 4.3, 4.4, 6.3, 6.5., 6.6, 8.3, 8.7

1. Which of the following implies that $\lim_{(x,y)\to(0,0)} f(x,y)$ exists?

a. $\lim_{x\to 0} f(x,0)$ exists b. $\lim_{y\to 0} f(0,y)$ exists

c. Both $\lim_{x\to 0} f(x,0)$ and $\lim_{y\to 0} f(0,y)$ exist, and $\lim_{x\to 0} f(x,0) = \lim_{y\to 0} f(0,y)$

d. $\lim_{(x,y)\to(0,0)\, Along\, C} f(x,y) = L$ for any line C : $y = mx$ e. None of the above f. All of the above

2. Which of the following implies that $f(x,y)$ is differentiable at (x_0, y_0)?

a. $f(x,y)$ is continuous at (x_0, y_0)

b. There exists a linear approximation $L(x,y)$ to $f(x,y)$ at (x_0, y_0)

c. Both partial derivatives of $f(x,y)$ exist near (x_0, y_0)

d. Error $f(x,y) - \left(f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)\right)$ approaches 0

3. Which of the following is **<u>NOT</u>** a true statement about $\nabla f(x_0, y_0)$?

a. If $f_x(x_0,y_0) = f_y(x_0,y_0) = 0$, then $\nabla f(x_0,y_0) = \vec{0}$

b. If $\nabla f(x_0,y_0) = \vec{0}$, then any directional derivative is 0 at (x_0, y_0)

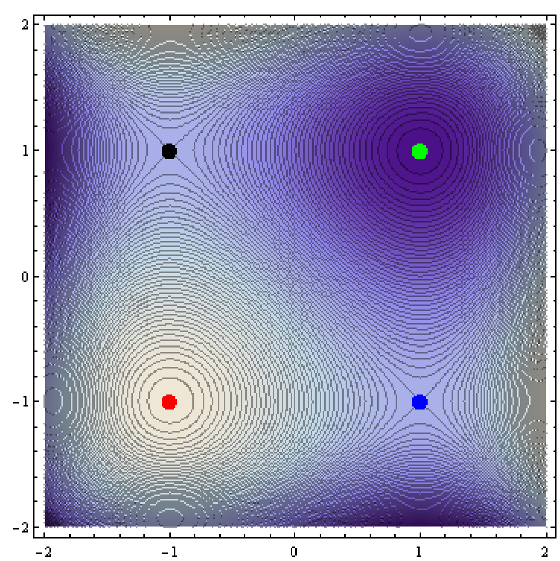
c. If $D_{\vec{v}}\, f(x_0,y_0) = D_{\vec{w}}\, f(x_0,y_0) = 0$ and $\vec{v} \neq k\,\vec{w}$, then $\nabla f(x_0,y_0) = \vec{0}$

d. If $\max_{\vec{v}} D_{\vec{v}}\, f(x_0,y_0) = 1$, then $\|\nabla f(x_0,y_0)\| = 1$

e. $\nabla f(x_0,y_0)$ is parallel to the level curve of $f(x,y)$ through (x_0, y_0)

4 Which of the following implies that a continuous on the region D function $f(x,y)$ attains an absolute maximum on D ? a. $D = \{x^2 + y^2 < 2\}$ b. $D = \{x + y > 2\}$ c. $D = \{xy \geq 0\}$ d. $D = \{0 \leq x, y \leq 5\}$

5. Function $f(x,y)$ has this contour plot :



a) How many critical points does $f(x,y)$ have?

b) Is there a local maximum? Where?

c) Is there a local minimum? Where?

d) Is there a saddle? Where?

e) At which saddle is $f(x,y)$ greater?

MATH 03 Chapter 14 Practice Test Please, ***show all work and circle final answers***.

Please, ***do NOT change*** e, $\sqrt{2}$, $\ln 3$ and so on, to decimals.

Please, use ***exact values*** for basic angles, for example, use $\frac{\sqrt{3}}{2}$ for $\sin\frac{\pi}{3}$.

1. Find the domain of the function: $f(x,y) = \ln(x^2 + y^2 - 4)$.

2. Sketch the contour plot of $f(x,y) = xy$.

3. Where is the vertex of a level curve of $f(x,y) = \frac{x-1}{2+(y+1)^2}$ located?

Find the limit (by a valid method) or prove that it does not exist.

4. $\lim_{(x,y)\to(2,1/2)} \frac{\arctan(xy)}{x-y}$.

5. $\lim_{(x,y)\to(2,-1)} \frac{x^2-4y^2}{x+2y}$.

6. $\lim_{(x,y)\to(0,0)} \frac{-1+\cos(x^2+y^2)}{x^4+2x^2y^2+y^4}$

7. $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

8. $\lim_{(x,y)\to(0,0)} xy\ln(x^2+y^2)$.

9. Where is the function $f(x,y,z) = \frac{x}{y\sqrt{z-x^2-y^2}}$ continuous?

10. Let $f(x,y) = \ln x + e^{xy}$. **a.** Find f_x **b.** Find f_{xy} **c.** Find $\frac{\partial^2 f}{\partial y^2}$

11. The area of a region in polar coordinates is $A(r,\theta) = 2r^3\cos\theta + \sin r\theta$, $\theta = \pi/6$ and $r = 2$.
    a. Find the rate of change of the area with respect to the polar angle
    b. The angle starts to increase at a rate of 0.05 radian per second and the distance starts to decrease 0.02 inches per second. At what rate is the area changing? Is it increasing?

12. The resistance R of two resistors in parallel is $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Use differentials to approximate the change in R if R_1 is increased from 4 to 4.1 ohms and R_2 is decreased from 7 to 6.8 ohms.

13. The height and radius of a right circular cylinder are 15 cm and 8 cm.
    a) The possible error in each measurement is ± 0.02 centimeters.
    b) The possible error in each measurement is 2.5%.
    Use the total differential to estimate the relative error in the calculated volume of the cylinder.

14. Use the appropriate Chain Rule to find $\frac{\partial z}{\partial v}$ for $z = \cos x \cos y$, $x = 2u + 3v$ and $y = u^2 - v^2$.

15. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ for $yz^2 + xz = 3x$.

16. Let $f(x,y,z) = x^2 - 2y^2 + z^2e^z$ and point P is $(-1,2,1)$.
    a. Find the gradient of f at P.
    b. Find the directional derivative of f at P in $\mathbf{v} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ direction.
    c. Find the maximum and minimum of directional derivative at P.

17. Find the path followed by a heat-seeking particle placed at the point (3, 4) on a plate with a temperature field $T(x,y) = 100 - 2x^2 - y^2$.

18. Find a unit normal vector to the level curve $f(x,y) = -2x^2 + 3y = 10$ at $(-1,4)$.

19. Find parametric equations for the normal line to $f(x,y) = 3yx^2 - y^2$ at $(-2,1,11)$.

20. Find the equation of the tangent plane at $(-2,0,3)$ to $18 = \mathrm{zy}^2 + \mathrm{xy}^2 - \mathrm{xz}^2 + 4\mathrm{y}$.

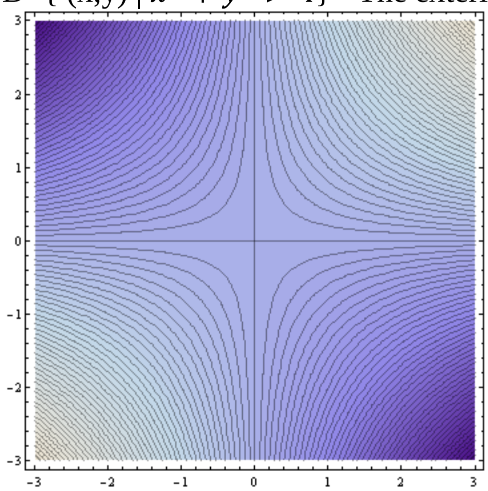
21. Use the Second Partials Test to determine the nature of the function f at point P, if $f_x(P) = f_y(P) = 0,\ f_{xx}(P) = 2,\ f_{yy}(P) = 8,\ f_{xy}(P) = 4$.
a. Relative min. b. Saddle point c. Relative max. d. Test is inconclusive

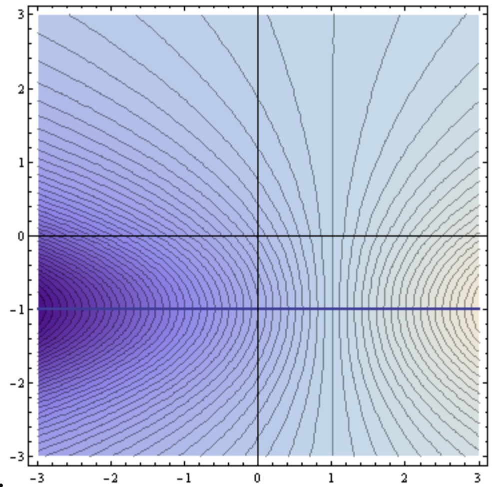
22. Find the relative extrema and saddle points of $f(x,y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$.

23. Find the absolute extrema of $f(x,y) = x + y$ on the disc $x^2 + y^2 \le 1$.

CHAPTER 14 PRACTICE TEST ANSWERS

1. D={ (x,y) | $x^2 + y^2 > 4$} - The exterior of the disk of radius 2.

2. 

3.  On $y = -1$ line

4. $\pi/6$. 5. 4 6. -1/2 7. DNE (If $x = 0$, $\frac{xy^2}{x^2+y^4} = 0$, If $x = y^2$, $\frac{xy^2}{x^2+y^4} = \frac{y^4}{2y^4} = \frac{1}{2}$)

8. 0 (switch to polar coordinates) 9. { (x,y,z) | $x^2 + y^2 < z$ and $y \neq 0$ }

10 a. $\frac{1}{x} + ye^{xy}$ b. $(1 + xy)e^{xy}$ c. x^2e^{xy}

11. a.-7 b. ≈-0.77 sq.in/sec, decreasing. 12. 0.01405. 13. a) ≈0.6% b)7.5%

14. $-3\sin(2u + 3v)\cos(u^2 - v^2) + 2v\sin(u^2 - v^2)\cos(2u + 3v)$

15. $\frac{3-z}{x+2yz}$. 16. a. -2**i** - 8**j** + 3*e***k** b. $-(26 + 6e)/\sqrt{14}$ c. $\sqrt{68+9e^2}$

17 $x = \frac{3y^2}{16}$ 18. $\pm\frac{4\mathbf{i} + 3\mathbf{j}}{\sqrt{5}}$ 19. $x = -2 - 12t,\ y = 1 + 10t,\ z = 11 - t$ 20. $9x - 4y - 12z = -54$.

21. d 22. Maximum:(0, 0, 4), Minimum:(2, 0, 0), Saddle points:(1, 1, 2) and (1, -1, 2).

23. Maximum: $f(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = \sqrt{2}$, Minimum: $f(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) = -\sqrt{2}$