Chapter 13 Concept Check

Find Answers in the book, pay special attention to

Theorems13: 2.8, 3.1, 3.3, 3.4, 5.2, 6.2, 6.3

1. Which of the following does **NOT** imply that
$\vec{r}(t)$ and $\vec{r}'(t)$ are orthogonal?

a. $\vec{r}(t) = \vec{c} = const$

b. $\|\vec{r}(t)\| = c = const$

c. $\|\vec{r}'(t)\| = c = const$

d. Tip of $\vec{r}(t)$ is always on the unit circle

2. Which of the following implies that
$\vec{r}(t)$ is arc length parametrization?

a. $\int \|\vec{r}'(t)\|\, dt = 1$ b. $\|\vec{r}'(t)\| = 1$ for all t

c. $\int_0^t \|\vec{r}'(u)\|\, du = 1$ d. $\|\vec{r}(t)\| = 1$ for all t

3. Which of the following does **NOT** imply that the curvature is zero?

a. $\frac{d\varphi}{ds} = 0$, where φ is the angle that plane curve makes with x-axis

b. $y''(x) = 0$ for a plane curve $y(x)$

c. $\vec{v}(t)$ and $\vec{a}(t)$ are parallel

d. $\vec{a}(t) = \vec{0}$

e. $\vec{v}(t)$ and $\vec{a}(t)$ are orthogonal

4. Which of the following does **NOT** imply that $a_T(t) = 0$?

a. $\vec{v}(t)$ and $\vec{a}(t)$ are orthogonal

b. $\vec{v}(t) = \vec{0}$

c. $\vec{a}(t) = \vec{0}$

d. $\|\vec{v}'(t)\|' = 0$

5. An object moves with constant speed of $\sqrt{14}$ along the circle of radius 7 centered at the origin. Find the vector normal component of the acceleration at the moment when the object is at the highest point.

a. $\sqrt{14}$ (**i** + **j**)

b. -7**j**

c. -2**j**

d. 2**i**

MATH 03 Chapter 13 Practice Test Please, ***show all work and circle final answers***.
Please, ***do NOT change*** e, π, $\sqrt{2}$, ln3, and so on, to decimals.
Please, use ***exact values*** for basic trig. angles, for example, use $\sqrt{3}/2$ for $\sin\pi/3$.

1. Find the domain of vector-valued function $\mathbf{r}(t) = 2\ln(3-t)\mathbf{i} + t^2\mathbf{j} + \ln t\ \mathbf{k}$.

2. Evaluate the limit: $\lim_{t\to 0}\left(\frac{e^{2t}}{t^2-1}i + \frac{1-\cos t}{t}j + \sqrt{4-t^2}k\right)$.

3. Calculate $\mathbf{r}'(t)$ for $\mathbf{r}(t) = \frac{1}{\sqrt{9t^2+5}}\langle 3t, 1, -2\rangle$.

4. Find $[\mathbf{v}(t)\times\mathbf{u}(t)]'$ if $\mathbf{v}(t) = t\mathbf{i} + t^2\mathbf{k}$ and $\mathbf{u}(t) = t^2\mathbf{j} - t\mathbf{k}$.

5. Evaluate: $\int(\sin 2t\,\mathbf{i} + 2\cos t\,\mathbf{j} + te^t\,\mathbf{k})\,dt$.

6. Sketch the curve represented by $\mathbf{r}(t) = 2\sin t\mathbf{i} + 3\cos t\mathbf{j}$ and sketch $\mathbf{r}(\pi/2)$ and $\mathbf{r}'(\pi/2)$ so that the initial point of $\mathbf{r}(\pi/2)$ is at the origin and the initial point of $\mathbf{r}'(\pi/2)$ is at the terminal point of $\mathbf{r}(\pi/2)$.

7. Let $\mathbf{r}(t) = e^{-t}(\mathbf{i} + \mathbf{j} + \mathbf{k})$. Calculate the unit tangent vector.

8. Find a set of parametric equations for the line tangent to the space curve $\mathbf{r}(t) = e^t\cos t\ \mathbf{i} + e^t\sin t\ \mathbf{j} + e^t\ \mathbf{k}$ at the point (1, 0, 1).

9. Find the principal unit normal vector for the curve represented by $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$ when $t=-1$.

10. Show that the binomial vector, $\mathbf{B} = \mathbf{T} \times \mathbf{N}$, is a unit vector.

11. Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$.
    a. Find $\mathbf{T}(t)$, $\mathbf{T}(1)$, and $\mathbf{N}(1)$.
    b. Sketch the graph of $\mathbf{r}(t)$.
    c. At the point $t = 1$, sketch the vectors $\mathbf{T}(1)$ and $\mathbf{N}(1)$.

12. a) Find the arc length of curve C: $\mathbf{r}(t)= 2t\mathbf{i}+t^2\mathbf{j}+(\ln t)\mathbf{k}$, $1 \le t \le 2$.

b) Find an equation in standard form of the normal plane to the graph of $\mathbf{r}(t)$ at P(2,1,0).

13. Find the curvature of the curve $\mathbf{r}(t)=\sqrt{2}\sin t\mathbf{i} + \cos t\mathbf{j} + \cos t\mathbf{k}$.

14. Sketch the function $y = \sin x$ and its circle of curvature at $x = -\pi/2$. Give the coordinates of the center of the osculating circle.

15. Find an arc length parameterization for $\mathbf{r}(t)= 3t^2\mathbf{i} + 2t^2\mathbf{j} - t^2\mathbf{k}$.

16. The vector-valued function $\mathbf{r}(t)= t^2\mathbf{i} + t^3\mathbf{j} + t\mathbf{k}$ describes the position of an object moving in space. Find its acceleration at $t=2$.

17. An object moves according to the position function $\mathbf{r}(t)= -2\cos 3t\ \mathbf{i} + \sin 3t\ \mathbf{j}$. Find the maximum speed of the object.

18. An object starts from rest at the point (0, 1, 1) and moves with an acceleration of $\mathbf{i} + \mathbf{j}$. Find the position at $t=4$.

19. Find the tangential and normal components of acceleration for the position function given by $\mathbf{r}(t)= t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$.

a) Scalar tangential

b) Scalar normal

c) Vector tangential

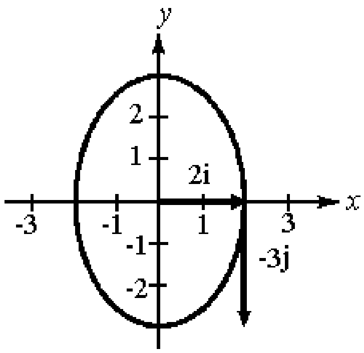
d) Vector normal

Answers to Calc3 Practice Test 2.

1. (0,3) 2. $-\mathbf{i} + 2\mathbf{k}$ 3. $3/(9t^2 + 5)^{3/2}\langle 5,-3t,6t\rangle$

4. $-4t^3\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}$ 5. $(-1/2)\cos 2t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (te^t - e^t)\mathbf{k} + \mathbf{C}$

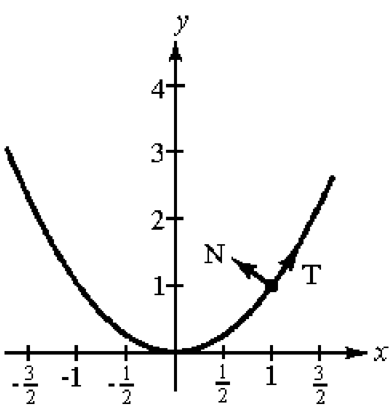
6.



7. $-1/\sqrt{3}\,(\mathbf{i} + \mathbf{j} + \mathbf{k})$ 8. $x=1 + t$, $y=t$, $z=1 + t$ 9. $1/\sqrt{5}\,(\mathbf{i} + 2\mathbf{j})$

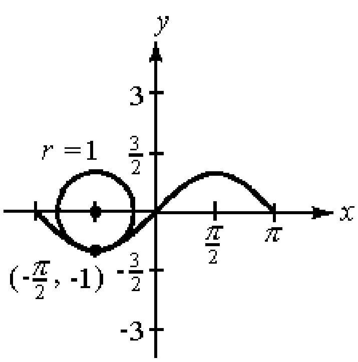
10. $\|B\| = \|T \times N\| = \|T\|\,\|N\|\sin\theta$, where θ is the angle between **T** and **N**. Because **T** and **N** are perpendicular, $\sin\theta = 1$. Hence, $\|B\| = (1)(1)(1) = 1$.

11. $\mathbf{T}(t) = 1/\sqrt{1+4t^2}\,(\mathbf{i}+2t\mathbf{j})$, $\mathbf{T}(1) = 1/\sqrt{5}\,(\mathbf{i}+2\mathbf{j})$, $\mathbf{N}(1) = 1/\sqrt{5}\,(-2\mathbf{i}+\mathbf{j})$



12. a) 3 + ln 2 b) 2x + 2y + z = 6 13. $1/\sqrt{2}$

14. Center $(-\pi/2,\ 0)$



15. $\dfrac{3s}{\sqrt{14}}\mathbf{i} + \dfrac{2s}{\sqrt{14}}\mathbf{j} - \dfrac{s}{\sqrt{14}}\mathbf{k}$ 16. $2\mathbf{i} + 12\mathbf{j}$ 17. 6 18. $8\mathbf{i} + 9\mathbf{j} + \mathbf{k}$

19. $a_T = 4t/\sqrt{2+4t^2}$, $a_N = 2/\sqrt{1+2t^2}$, $a_T\mathrm{T} = \dfrac{2t}{1+2t^2}(\mathbf{i}+\mathbf{j}+2t\mathbf{k})$, $a_N\mathrm{N} = \dfrac{2}{1+2t^2}(-t\mathbf{i} - t\mathbf{j} + \mathbf{k})$